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A Mathematical Model for n-Species Competition for Aphid Population in Limited Resources

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ABSTRACT

In this paper, a mathematical model is discussed for the competition in to n-species aphid population for a limited resource and their limiting behavior as time approaches infinity. Aphids are among the most conspicuous and important pests in the green houses and the fields. Aphids which excrete honeydew and the area covered by saliva $\int_0^t n(s)ds$, *at time* t, *which forms a weak cover on the leaf surface preventing the individuals from further sucking and movement and so causing deaths due to starvation, therefore rapid increase of mortality rate to the environment by honeydew excretions of aphids. Analytically, it has been shown that how aphid population increases or decreases with time 't'.*

Keywords: Aphids, Environment, Population dynamics, Limited resources, Voltra model, $n-$Species

INTRODUCTION

Aphids are plant lice small, soft-bodied sap-sucking insect pests in size from 1-7 mm with long legs and antennae. Aphids are most destructive insect pests have a wide host range significantly affect various field crops, fruit and vegetables as sucking pests (Aheer et.al.2008) Aphids are generally divided in to two sub-groups known as green-flies and black-flies and they contain different type of colors such as is yellow, pink, and white etc. Generally, more than five hundreds species of aphid population has been found in east part of India. Aphids discharges lethal mucilaginous material, called saliva, smeared over the surface of the leaf preventing the individuals from further sucking and movements causing deaths due to hungry. Some species inject their saliva in to plant roots and slowly damaged the plants and attack a wide range of plant hosts. Some species when moving from one plant to other they spread out plant virus diseases, which has been seen in some soft fruits, such as strawberry and some vegetables such as tomatoes, beets, brinjals, sweet peas, etc. in a similar manner as in wheat (Jarosik et.al. 2003; Ahamad et.al. 2006). Some aphid species attack different parts of plants other than leaves. The behavior of beet root aphid penetrate in to the soil and attacks beet roots during most of their life, causing beet plants to faccid and occasionally die if growth rate of aphid population is very high. Aphids are easily driven to death when weather, is not according to their survival. Aphids are infertile in extreme temperature because due to higher temperature their symbiotic bacteria are kills on which some aphids are dependent (Lamb et.al. 2013) Various attempts have been made to develop the mathematical models of their population dynamics (Barlow et.al. 1982; Barlow et.al.1980) In experimental observation by (Okrouhla.1983), it was seen that total number of individuals in the population of *Aphis fabae* rapidly increased at the beginning, reaching its maximum on the 5^{th} day and then decreased rapidly as a high consequence of mortality of the aphids. The first and the simplest law of population growth of a species was given by (Malthus.1798) as

$$\frac{dP}{dt} = kP \qquad (1)$$

From which we get

$$P = P_0 e^{kt} \qquad (2)$$

where $P(t)$ denotes the population at time t, P_0 is the population at time $t = 0$ and k is growth rate of population. The Malthusian growth equation implies that $P \to \infty \ as\ t \to \infty$ as (Pearl.1925) reasoned crowding affects due to excessive demands on a limited food supply. He shows that fecundity decreases and even raising deaths due to starvation. It is assumed that due to crowding, the growth rate k is reduced by some proportion of the population P. Thus, the equation of the population growth of a single species is given by

$$\frac{dP}{dt} = (k - bP)P \qquad (3)$$

The population P asymptotically approaches to a limiting value $\frac{k}{b}$ for $k > 0\ as\ t \to \infty$.
In these mathematical models, it is assumed that the population consists of one species only a rather isolated phenomenon in the nature .A number of species are competing for a limited resource and the population density of one species may affect the growth rate of the other. In view of the above we have developed a mathematical model for n- species competition for aphid population on plant and their limiting behavior for limited resources as time approaches to infinity under a particular set of environmental conditions. In the regard, (Meihls et al.2008) studied the population growth of soya been, *aphin glyeines* under varying labels of predator exclusion.

n- Species competition model

When internal competition in a species for limited resources of aphid population are taken into account the n-species competition model is given as (Volterra. 1927 ; Kapur. 1989)

$$\frac{dP_r}{dt} = \left(k_r - \sum_{s=1}^{n} b_{rs} P_s\right) P_r k P_r \int_0^t P_r(s) ds \qquad (4)$$

Where , $r = 1,2.3 \ldots n$ and k is Positive constant.
where P_r denotes the population at time t of r^{th} species $k_r > 0$ is the r^{th} species growth rate in the absence of other species, $b_{rs} > 0$ is the effect of the density of s^{th} species on the population growth rate of r^{th}species.
To get the solution of non linear differential equation (4) we assume that there are some environmental conditions under which the effect of density of one species on the growth rate of other species is the same as the effect of the density of a species on its own growth rate. From the equation (4) we get-

$$\frac{dP_1}{dt} = (k_1 - b_{11}\bar{P})P_1 - kP_1 \int_0^t P_1(s)\, ds$$

$$\frac{dP_2}{dt} = (k_2 - b_{22}\bar{P})P_2 - kP_2\int_0^t P_2(s)ds$$

$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$

$$\frac{dP_n}{dt} = (k_n - b_{nn}\bar{P})P_n - kP_n\int_0^t P_n(s)ds$$

Where $\bar{P} = P_1 + P_2 + \cdots + P_n$

Now we shall consider two type species i and j and investigate their competitive result

Thus we can take-

$$\frac{dP_i}{dt} = (k_i - b_{ii}\bar{P})P_i - kP_i\int_0^t P_i(s)ds$$

and

$$\frac{dP_j}{dt} = (k_j - b_{jj}\bar{P})P_j - kP_j\int_0^t P_j(s)ds$$

Le us put $N(t) = \int_0^t P_i(s)ds$

and $M(t) = \int_0^t P_j(s)ds$

$$\frac{dP_i}{dt} = (k_i - b_{ii}\bar{P})P_i - kP_iN(t) \tag{5}$$

and

$$\frac{dP_j}{dt} = (k_j - b_{jj}\bar{P})P_j - kP_jM(t) \tag{6}$$

Multiplying (5) and (6) by $\frac{1}{b_{ii}P_i}$ and $\frac{1}{b_{jj}P_j}$ respectively then subtract we have- $\frac{1}{b_{ii}P_i}\frac{dP_i}{dt} - \frac{1}{b_{jj}P_j}\frac{dP_j}{dt}$

$$= \frac{k_i}{b_{ii}} - \frac{k_j}{b_{jj}} k\left\{\frac{N(t)}{b_{ii}} - \frac{M(t)}{b_{jj}}\right\}$$

or

$$\frac{d}{dt}\left\{log\left(\frac{P_i^{\frac{1}{b_{ii}}}}{P_j^{\frac{1}{b_{jj}}}}\right)\right\} = c_{ij} - k\left\{\frac{N(t)}{b_{ii}} - \frac{M(t)}{b_{jj}}\right\} \tag{7}$$

where, $\frac{k_i}{b_{ii}} - \frac{k_j}{b_{jj}} = c_{ij}$

Then the integration of the above differential equation leads to

$$log\left(\frac{P_i^{\frac{1}{b_{ii}}}}{P_j^{\frac{1}{b_{jj}}}}\right) = c_{ij}t - k\left\{\frac{N(t)}{b_{ii}} - \frac{M(t)}{b_{jj}}\right\}t + logM$$

Where M is the constant of integration, its value is given by

$$\frac{P_i^{\frac{1}{b_{ii}}}}{P_j^{\frac{i}{b_{jj}}}} = Me^{c_{ij}t-k\left\{\frac{N(t)}{b_{ii}}-\frac{M(t)}{b_{jj}}\right\}t+logM}$$

$$\frac{(P_i)_0^{\frac{1}{b_{ii}}}}{(P_j)_0^{\frac{1}{b_{jj}}}} = M \qquad (8)$$

where $(P_i)_0$ and $(P_j)_0$ are the value of P_i and P_j and, $N(0) = 0 \; = M(0)$, at time $t = 0$ that is, the initial stage of observation period.

RESULTS AND DISCUSSION

From equation (8), we note that there are three cases arise here,

Case I. If $c_{ij} = 0$ then $P_i^{\frac{1}{b_{ii}}} = MP_j^{\frac{1}{b_{jj}}}$, for every $t > 0$, thus we see that the ratio of population of two species remains constant with the passage of time.

CaseII. If $c_{ij} > 0$,then $\frac{P_i^{\frac{1}{b_{ii}}}}{P_j^{\frac{i}{b_{jj}}}} = \infty \; as \; t \to \infty$

This implies that $P_{j \to 0}$, thus we may say that the j^{th}species become extinct.

Case III. If $c_{ij} <0$,then $\frac{P_i^{\frac{1}{b_{ii}}}}{P_j^{\frac{i}{b_{jj}}}} = 0 \; as \; t \to \infty$

This implies that $P_i \to 0$, thus we may say that i^{th}species becomes extinct

.

CONCLUSION

If we identify the i^{th}-th species as j^{th}species and j^{th}species as the i^{th}species then the competition tensor c_{ij} helps us to predict the behavior of the species of aphid population as the time approaches infinity. From (7), it can easily be seen that the competition tensor c_{ij} is a skew- symmetric tensor (since $c_{ij} = -c_{ji}$). In view of case I, II, and III, we see that if any of the quantity $c_{i1}, c_{i2} \dots c_{in}$ is negative, the i^{th} species goes to extinction. We can then find the i^{th} row from the matrix $[c_{ij}]_{n\times n}$, the density of surviving species then will be the column number of the negative $c_{i1}, c_{i2}, \ldots c_{im}$. The interaction of aphid species continuous, till we are left with one row.

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