Available online at www.scientiaresearchlibrary.com

Scientia Research Library **ISSN 2348-0416**
USA CODEN: JASRHB

Journal of Applied Science And Research, 2014, 2 (2):1-13

(*http://www.scientiaresearchlibrary.com/arhcive.php*)

TO THE PROBLEM OF MHD WAVES PROPAGATION IN THE IONOSPHERIC E- REGION

George Jandieri
Georgian Technical University, Georgia

ABSTRACT
The dispersion equation has been obtained describing propagation of very slow and long-period MHD waves in the ionospheric E-region. Statistical characteristics of the low-frequency MHD waves propagating in weakly ionized plasma are obtained for arbitrary correlation function of the particles density fluctuations. Energy exchange between "fast" and "slow" Alfven waves and the turbulent plasma flow is analyzed in the ray (optics) approximation using the stochastic eikonal equation. Correlation function of the phase fluctuations and the broadening of the temporal spectrum of scattered Alfven wave are calculated numerically for the anisotropic correlation function using the experimental data.
Keywords: Alfven wave, plasma flow, statistical characteristics, broadening, irregularities.

INTRODUCTION

The wavy processes in the upper atmosphere have both, hydrodynamic and electromagnetic nature. In the first class of waves belong the acoustic (sonic), gravitational and MHD (Alfven and magnetoacoustic) waves, while the second class of waves contains planetary Rossby waves and magnetogradient waves [1]. General dispersion equation was derived for the magneto-acoustic, magneto-gravity and electromagnetic planetary waves in the ionospheric *E*- and *F*-regions [1,2]. The geomagnetic field generates small and medium-scale waves: magneto-acoustic and Alfven waves. In the ionosphere magneto acoustic waves are generated by the elasticity of the geomagnetic lines of force. These are fast (with the propagation velocity more than 1 $\mathrm{km\cdot s^{-1}}$ and short-period (of the order of $5-20$ min). Alfven waves, with phase velocity depending on the orientation of the wave vector $\mathbf{k}$ with respect to the geomagnetic field, $\mathbf{H_0}$ are generated due to the tension of the geomagnetic lines of force and as it will be shown below can be very slow ($10\div50$ $\mathrm{m\cdot s^{-1}}$) and long-period ($1\div2$ days), when the wave vector $\mathbf{k}$ is almost transversal to $\mathbf{H_0}$ and fast, when vectors $\mathbf{k}$ and $\mathbf{H_0}$ are parallel.

The ionospheric observations reveal the electromagnetic perturbations in the *E* – region known as the slow MHD waves [3,4]. These waves are insensitive to the spatial inhomogeneities of the Coriolis and Ampere forces and are propagated in the ionospheric medium more slowly than the ordinary MHD waves. In natural conditions, these perturbations are revealed as background oscillations [1]. Observations show [5-7] that during earthquakes, man-made explosions, magnetic storms, launching of space crafts, worldwide networks of ionospheric and magnetic observatories (located approximately along one latitude) in the *E*-region ($70 \div 150$ km) of ionosphere besides the well-known wave modes the large-scale ($\lambda \sim 10^3 \div 10^4$ km) ionospheric wave disturbances of electromagnetic nature are clearly registered propagating along the parallel around the Earth with high (supersonic) speeds (higher than $1\,\mathrm{km \cdot s^{-1}}$) and having periods from several minutes to several hours. In the same region of the upper atmosphere are also observed large-scale, long-period (from two days to two weeks and longer) wave disturbances of hydrodynamic nature (their velocity is of the order of the velocity of the ionospheric winds). Unlike the very long planetary Rossby waves (propagating mainly westward) they propagate to the east and generating the electric currents lead to the essential pulsations of the geomagnetic field (to 45 nT and higher).

According to the numerous observations [4] at the ionospheric moderate and high latitudes large-scale (up to 10^3 km), and long-period (with characteristic time scale of 0.5-2 hours) ionospheric wavy perturbations regularly exist which are propagating zonally over long distances (to ten thousand of kms) with the velocity more than 1 $\mathrm{km \cdot s^{-1}}$. The observed propagation velocity of wave cannot be explained within the frames of hydrodynamic theory of ordinary acoustic gravity waves, since the maximum characteristic velocity of the letter, at the ionosphere altitudes does not exceed 700-800 $\mathrm{m \cdot s^{-1}}$. The velocities of the order of 1 $\mathrm{km \cdot s^{-1}}$ and more are arising when the influence of partial "freezing-in" of the geomagnetic field on the propagation of MHD waves in the ionosphere is taking into account. For the *E*-region the plasma component behaves like a passive impurity. The neutrals completely drag ions and the "ionospheric" friction between neutrals and ions can be neglect [1]. Therefore velocity of the neutral component $H_0 / \sqrt{4\pi M N_n}$ is much lower than velocity of the plasma component $H_0 / \sqrt{4\pi M N}$, where N_n and N denote concentrations of the neutral particles and charged particles of the ionospheric plasma, respectively. Below we consider slow, long period, large-scale MHD waves in *E*- layer of the ionosphere.

The features of low-frequency waves in homogeneous magnetized plasma are well studied [8], however little attention is devoted to the investigation of statistical characteristics of MHD waves in turbulent plasma flow observing in both cosmic and laboratory conditions. It was established [9] that statistical moments of these waves substantially depend on a type of waves. Therefore propagation of MHD waves in the turbulent plasma streams is of practical interest. Some peculiarities of statistical characteristics of MHD waves in turbulent plasma using the "freezing-in" turbulence approximation have been considered [10].

Statistical characteristics of the "fast" and "slow" Alfven waves in the *E*-region of ionosphere with randomly varying spatial-temporal plasma parameters are considered in this paper. The energy exchange between Alfven wave and turbulent plasma flow is analyzed calculating the mean energy flux density. Numerical calculations have been carried out using the experimental data.

SMALL OSCILLATIONS OF THE EARTH'S IONOSPHERE

Linearized equation of motion of the MHD set of equations describing wavy processes in the ionosphere taking into account Hall's effect has the following form

$$\frac{\partial \mathbf{V}}{\partial t} = \frac{1}{\rho c}[\mathbf{j} \cdot \mathbf{H}] - \frac{1}{\rho} grad\, P + \mathbf{g} + [\mathbf{V} \cdot 2\boldsymbol{\omega}_0], \tag{1}$$

where P and $\rho = \rho_n + \rho_{pl} \approx \rho_n = M\,N_n$ are pressure and density of the neutral particles; M is mass of ions (molecules); V and $\mathbf{H}$ are vectors of the fluid velocity and magnetic field, respectively; g is the vector of gravitational acceleration, $\boldsymbol{\omega}_\mathbf{0}$ is the angular velocity of the Earth's rotation, $\mathbf{j}$ is the current density, c is the speed of light, $\mathbf{F_A} = [\mathbf{j} \cdot \mathbf{H}] / c\rho$ is the electromagnetic Ampere's force [1]. In numerous observations [11,12] the real atmosphere quickly restores the violation of both quasi-static (during several minutes) and quasi-geostrophic (roughly during one hour) states. Thus, for the synoptic processes (two weeks and more), it may be considered that the atmosphere always is in the quasi-static and quasi-geostrophic states. It was shown [13] that neutral Rossby waves and acoustic-gravity waves are eliminated if the quasi-static and quasi-geostrophic conditions are fulfilled in the atmosphere. In this case, separating electromagnetic effects of slow MHD waves and neglecting all hydrodynamic forces Equation [1] reduces to: $\partial \mathbf{V} / \partial t = [\mathbf{j} \cdot \mathbf{H_0}] / \rho c$, $\mathbf{H} = \mathbf{H_0} + \mathbf{h} \approx \mathbf{H_0}$. In the upper layers of the E-region (at altitudes of 100-150 km), fast waves with a wavelength of 2000 km are significantly damped due to the Pedersen conductivity. However, longer waves are damped weakly. Therefore, in this paper, in considering long-wavelength perturbations with $\lambda \sim 10^4$ km we neglect, for simplicity, the Pedersen conductivity in the Hall layer, we assume that $\mathbf{H_0} = H_{0z} \mathbf{e_z}$. Restricting ourselves by moderate and high latitudes (geomagnetic field has only vertical component H_{0z}), generalized Ohm's law for the E-region can be expressed in the following form

$$\frac{1}{c}[\mathbf{j} \cdot \mathbf{H}] = e\,N\left(\mathbf{E} + \frac{1}{c}[\mathbf{V} \cdot \mathbf{H}]\right). \tag{2}$$

Assuming the equality $[\mathbf{j} \cdot \mathbf{H_0}] = 0$ [1, 4], we get:

$$\mathbf{E} = -\mathbf{w} - i\,g\,[\mathbf{w} \cdot \boldsymbol{\tau}]\ , \qquad \mathbf{j} = -\frac{c^2}{4\pi V_a^2} \frac{\partial \mathbf{w}}{\partial t}\ . \tag{3}$$

where $\mathbf{w} = [\mathbf{V} \cdot \mathbf{H_0}] / c$ is the dynamo field caused by the wind mechanism, $\boldsymbol{\tau} = \mathbf{H_0} / H_0$ is the unit vector along the strength of the geomagnetic field, $g = \omega / \Omega_i$ is the ratio of the wave frequency to the ion gyrofrequency, $\Omega_i = \eta\,\omega_i$ is modified by the ionization degree cyclotron frequency of ions (in the E-region Ω_i is of the order of $10^{-4} \div 10^{-5}\ \mathrm{s}^{-1}$) (1), $\omega_i = e\,H_{0z} / M\,c$ is the cyclotron frequency of ions ($\omega_i \approx 10^2\ \mathrm{s}^{-1}$). In the neutral component of the ionosphere the velocities of the order of 1 $\mathrm{km} \cdot \mathrm{s}^{-1}$ and they are insignificant for MHD waves in the plasma component ($\sim 10^3\ \mathrm{km} \cdot \mathrm{s}^{-1}$). This is stipulated by the fact that in the ionosphere for the long-period processes the geomagnetic field is "frozen" into the plasma component and during the perturbations it passes its perturbation to the neutral component by collision processes. In the neutral part, it propagates with the Alfven velocity $V_a = H_0 / \sqrt{4\pi\rho} = H_0 / \sqrt{4\pi M\,N_n} = \sqrt{\eta}\,V_A$, where $V_A = H_0 / \sqrt{4\pi M\,N}$ is the velocity of the MHD wave in the plasma component of ionosphere. In the ionospheric E (70-150 km, $\omega_i / \nu_{in} \sim 10^{-2} << 1$, ν_{in} is the collision frequency between ions and neutral particles) and F (150-600 km) regions the ionization degree $\eta = N / N_n$ is of the order of $10^{-8} \div 10^{-4}$. Therefore, the value of V_a is much less than V_A. Consequently we naturally come to the consideration of slow (in the electrodynamics sense) long-period MHD waves in the ionosphere [1]. In the E-region $V_a = 1 \div 2\ \mathrm{km} \cdot \mathrm{s}^{-1}$(if geomagnetic field is 0.2 Gauss).

Substituting equation (3) into Maxwell's equation $rot\,rot\,\mathbf{E} = -(4\pi / c^2)\,\partial\,\mathbf{j} / \partial t$, at $\mathbf{w} \sim \exp(i\,k_x x + i\,k_z z - i\,\omega t)$ we obtain the wave equation:

$$\frac{\partial^2 \mathbf{w}}{\partial t^2} + V_a^2\, rot\, rot\, \mathbf{w} = i\, g\, V_a^2\, rot\, rot[\mathbf{w}\cdot\boldsymbol{\tau}] \ , \tag{4}$$

The last term takes into account the Hall's effect. If $V_z = 0$, $V_x \neq 0$, $V_y \neq 0$, we obtain set of algebraic equations, and hence, the dispersion equation:

$$(\omega^2 - V_a^2 k^2)(\omega^2 - V_a^2 k_z^2) = g^2 V_a^4\, k^2 k_z^2 \ , \tag{5}$$

where: $k^2 = k_x^2 + k_z^2$. Equation (5) (in the electrodynamics sense) describes very slow and long-period (from two days to two weeks and more) MHD waves in the ionospheric *E*-region. The first bracket describes propagation of transverse Alfven wave in the ionosphere. Compressibility and stratification of the ionosphere do not play any role in the Alfven's waves and, therefore, for these waves the transversality condition $[\mathbf{k}\cdot\mathbf{V}] = 0$ is always satisfied [1]. At large k_x, if $k_z^2 << k_x^2$ and $V_a k_z << \omega$, from Equation (5) we get:

$$\omega^2 = V_a^2 k_x^2 \left(1 + \frac{V_a^2 k_z^2}{\Omega_i^2} \right) . \tag{6}$$

From Equation (6) it follows that in the *E*-region the characteristic horizontal wavelength $\lambda_0 = 2\pi V_a / \Omega_i$ exists, which determines the characteristic "length of dispersion" caused by the Hall's effect. If $V_a k_z << \Omega_i$ frequency of magneto-acoustic wave $\omega_M = V_a k_x$ increases linearly with k_x; if $\Omega_i << V_a k_x$ wave frequency is subject to the frequency of helicons ω_h:

$$\omega = \omega_h = k_x k_z \frac{V_a^2}{\Omega_i} = k_x k_z \frac{c H_{0z}}{4\pi N e} . \tag{7}$$

In ionospheric physics, they are known as "atmospheric whistlers". As a result, helicons in the *E*-region are the limiting case of magnetic sound. In helicons only electrons of the ionospheric plasma are oscillating together with the frozen in geomagnetic field lines [1]. At small k_x, if $k_z^2 >> k_x^2$ and $\omega < \Omega_i$ from Equation (5) we obtain frequency of the Alfven wave $\omega_A = V_a k_z$. For the second root, taking into account Equation (5), at $k_z^2 << k_x^2$ and $\omega << V_a k_x$ we get

$$\omega^2 = V_a^2 k_z^2 \frac{\Omega_i^2}{\Omega_i^2 + V_a^2 k_z^2} , \tag{8}$$

At $\Omega_i >> V_a k_z$ expression (8) describes Alfven wave with dispersion $\omega = V_a k_z$. At big wavenumber k_z the wave frequency is subject to the characteristic frequency $\omega \to \Omega_i = \eta\, \omega_i$. Consequently, waves Ω_i in the ionosphere are the limiting case of the quasi-transversal very low-frequency Alfven waves.

If $k_z^2 >> k_x^2$ from Equation (5) we obtain new branch of the modified Alfven wave having frequency: $\omega_{A*} = V_a^2 k_z^2 / \Omega_i = V_a^2 k_z^2 / \eta\, \omega_i$ and the ordinary Alfven wave: $\omega = V_a k_z$. For $k_x = 0$ from Equation (5) we obtain the dispersion relation for the Alfven-type waves propagating with phase velocity $V_{ph\,A}^2 = V_a^2 / (1 \pm \omega_h / \omega)$. For the frequencies $\omega >> V_a k_z$ and $k_z > k_x$ phase velocity of helicon is: $V_{ph} = c H_{0z} k_z / 4\pi e N$. In absence of dispersion (Hall effect), for magneto-acoustic and Alfven waves we obtain: $V_{ph\,M} = H_{0z} / \sqrt{4\pi M\, N_n} = V_a$ and $V_{ph\,A} = V_a\,\theta$, where $\theta = k_z / k_x$. Slow magneto-acoustic waves propagate with the velocity of Alfven waves in the direction perpendicular to the external magnetic field. Phase velocity of Alfven waves depends on an angle $\theta << 1$. Using typical values of concentration of the neutral components N_n in the *E*-layer of ionosphere for large-scale magneto-acoustic waves we obtain $V_{ph\,M} = V_a = 1 \div 2\ \mathrm{km \cdot s^{-1}}$. For slow planetary waves with the period of $T_0 = 2$ days, horizontal wavelength $\lambda_x \approx 3000$ km, $k_x = 10^{-6}\ \mathrm{m^{-1}}$ and the wave number $k_z = 2\pi / T_0 V_a$ we get $\sim 10^{-8}\ \mathrm{m^{-1}}$, $\theta \approx 10^{-2}$.

Hence, the phase velocity of large-scale, slow Alfven waves is of the order of $V_{ph\,A} = 20\ \mathrm{m \cdot s^{-1}}$. For other periods T_0 and the horizontal wavelengths of planetary waves, the phase velocity of slow Alfven type waves does not exceed typical values of the wind velocities in the *E*-layer of ionosphere (from several meters per sec up to $100 \div 300\ \mathrm{m \cdot s^{-1}}$). These waves propagate transversally to the external magnetic field H_{0z} $(\theta << 1)$ and the total wave vector $\mathbf{k}$ is almost horizontal. Taking into account dispersion (Hall's effect), planetary waves are circularly polarized. For typical values of the wind velocity V in the *E*-region of ionosphere using the formula $h \approx H_{0z}\, V / V_{ph\,A}$ we can estimate perturbation of the geomagnetic field h varying within the range of 15-50 nT. The obtained results are in agreement with experimental data confirming the existence of planetary waves with velocities of 20-100 $\mathrm{m \cdot s^{-1}}$) in E -layer of ionosphere [15, 16] in any season of year with wavenumber of 2-10 $\mathrm{m^{-1}}$. In contrast with the ordinary Rossby waves, they lead to the substantial distortion of the geomagnetic field (from several to several tens of nT) revealing electromagnetic character of these waves. Large-scale disturbances (with the velocity of 1-2 $\mathrm{km \cdot s^{-1}}$) can be identified with magneto-acoustic waves in the neutral component of the ionosphere, and the planetary waves with the velocity of the motion 20-100 $\mathrm{m \cdot s^{-1}}$ can be identified with slow Alfven waves.

Hence, in the E region of ionosphere are exist: magneto-acoustic wave with frequency $\omega_M = V_a\, k_x$, Alfven wave with frequencies ω_{A*} and $\omega_A = V_a\, k_z$, slow cyclotron wave of ions Ω_i and helicons with frequency ω_h.

Wave equation for the electric field $\mathbf{E}$ with the current density $\mathbf{j}$ (Equation (3)) has the following form: $rot\,rot\,\mathbf{E} - k_0^2\,\mathbf{E} = -\,k_0^2\, c^2\, \mathbf{w} / V_a^2$. Multiplying Equation (3) scalar and vector on vector $\boldsymbol{\tau}$ we obtain:

$$\mathbf{w} = \frac{1}{b}\left\{ \mathbf{E} - g^2\,(\mathbf{E}\cdot\boldsymbol{\tau})\,\boldsymbol{\tau} - i\,g\,[\mathbf{E}\cdot\boldsymbol{\tau}] \right\}, \tag{9}$$

where: $b = g^2 - 1$.

Electric induction linearly connecting with the vector $\mathbf{w}$, $\mathbf{D} = -\,c^2\,\mathbf{w} / V_a^2$, allow to calculate components of the permittivity tensor of magnetized plasma describing slow MHD waves in the coordinate system when Z-axis is directed along the line of forces of geomagnetic field $\mathbf{H_0}$, $E_z = 0$, which readily yield:

$$\varepsilon_{xx} = \varepsilon_{yy} = \frac{c^2}{V_a^2}\frac{1}{1-g^2}, \quad \varepsilon_{xy} = -\,\varepsilon_{yx} = i\,\frac{c^2}{V_a^2}\frac{g}{1-g^2}, \quad \varepsilon_{xz} = \varepsilon_{yz} = \varepsilon_{zx} = \varepsilon_{zy} = 0, \quad \varepsilon_{zz} = \infty.$$

These expressions at $\eta=1$ have been obtained in [17] on the bases of the equations of two-fluid hydrodynamics of cold plasma taking the ion inertia into account but neglect the electron inertia and particle collision. Features of low–frequency waves phenomena in homogeneous magnetized plasma have been studied in [8,18].

On the other hand using the expression $\mathbf{D}=\varepsilon\mathbf{E}=4\pi i\,\mathbf{j}/\omega$ (19) (here ε is scalar) and substituting Equation (3) we get:

$$\left(\varepsilon-\frac{c^2}{V_a^2}\right)\mathbf{w}=-i\,g\,\varepsilon\,[\mathbf{w}\cdot\boldsymbol{\tau}] \,. \tag{10}$$

Multiplying this equation scalar and vector on vector $\boldsymbol{\tau}$ and taking into account $(\mathbf{w}\cdot\boldsymbol{\tau})=0$ we yield $\varepsilon=c^2/V_a^2(1\pm g)$. Transversal wave propagates in medium with the velocity $\omega/k=c/N$, where $N=\sqrt{\varepsilon_\perp}$ is the refractive index. Hence phase velocity of transversal MHD wave is $V_{ph}^2=V_a^2(1\pm g)$. This means that fast and slow Alfven waves are circularly polarized due to Hall's effect.

SECOND ORDER STATISTICAL MOMENTS OF THE ALFVEN WAVE IN A TURBULENT PLASMA FLOW

From the dispersion equation (5) follows that frequency of Alfven wave propagating along the external magnetic field is defined as $\omega=\pm V_a\,k_z$, upper and lower signs correspond to the "fast" Alfven wave (FAW) and "slow" (SAW) Alfven wave, respectively. Let small amplitude E_0 low frequency $\omega_0<<\omega_i$ monochromatic plane wave with the wave vector $\mathbf{k_0}$ generating in the $Z=0$ plane propagates along the Z - axis. Alfvén velocity exceeds thermal velocities of particles of low pressure plasma. Let's turbulent plasma flow with the velocity $\mathbf{V_0}$ moves along the external magnetic field $\mathbf{B_0}$ locating in the *XZ* plane (principle plane) of the Cartesian coordinate system with the angle of inclination θ with respect to the *Z*-axis. Using the eikonal equation the dispersion equation of the Alfven wave in the turbulent plasma flow has the following form [10] $\omega-(\mathbf{k}\,\mathbf{V_0})=\pm V_a\,(\mathbf{k}\,\mathbf{b})$; $\mathbf{b}$ is the unit vector along an external magnetic field; We suppose that in turbulent ionospheric plasma density fluctuations of the neutral particles exceed velocity pulsations $V_1/V_a<<N_1/N_n<<1$ ($V_1(\mathbf{r},\,t)$ represents small turbulent pulsations of the macroscopic velocity of the plasma flow). Frequency and wave number of the Alfven wave in turbulent plasma with smooth spatial-temporal fluctuations in the ray- (optics) approximation [20] satisfy the conditions, i.e. $k_0\,l>>1$, $\omega_0\,T>>1$ and $\omega_0\,l/V_0>>1$ (l and T are characteristic spatial-temporal scales of irregularities, the mean velocity of a plasma flow $\mathbf{V_0}$ is constant). For low frequency waves $\omega_0<<\omega_i$ and $V_a/c<<1$ the features of normal waves is the same as in one-liquid MHD approximation [18]. Neutral particles velocity and density can be expressed as sum of the regular and fluctuating components which are slowly varying random functions of the spatial coordinates and time $\mathbf{V}(\mathbf{r},t)=\mathbf{V}_0+\mathbf{V}_1(\mathbf{r},t)$, $N_n(\mathbf{r},t)=N_0+N_1(\mathbf{r},t)$. Substituting the wavevector $\mathbf{k}(\mathbf{r},t)=-\nabla\varphi$ and the frequency $\omega(\mathbf{r},t)=\partial\varphi/\partial t$ in the dispersion equation of the Alfven wave, taking into account that the phase is a sum of the regular $\varphi_0=\omega_0\,t-k_0\,z$ and fluctuating phases, $\varphi(\mathbf{r},t)=\varphi_0(\mathbf{r},t)+\varphi_1(\mathbf{r},t)$ ($\varphi_1<<\varphi_0$) we obtain stochastic transport equation for the phase fluctuation [10]:

$$\frac{\partial\varphi_1}{\partial t}+(\mathbf{V_{gr}}\cdot\nabla\varphi_1)=\mp\frac{1}{2}\,k_0\,V_{a\,0}\cos\theta\,\frac{N_1}{N_0}, \tag{11}$$

where: $\mathbf{V_{gr}} = (V_0 \pm V_{a\,0})\,\mathbf{b} = V_*\,\mathbf{b}$ is the group velocity for both Alfven waves, $V_{a\,0} = H_0 / \sqrt{4\pi M\, N_{n\,0}}$. This equation easily solved using the method of characteristics [10]:

$$\varphi_1(\mathbf{r},t) = p \int_0^L dz'\, N_1(x', y, z', t'), \qquad (12)$$

where: $x' = x - (z - z')\, tg\theta$, $t' = t - (z - z') / V_* \cos\theta$, $p = \mp\, \omega_0 V_{a\,0} / 2\, N_0 V_*^2 \cos\theta$. Correlation function of the phase fluctuations of scattered Alfven wave in the turbulent plasma flow has the following form:

$$V_\varphi^{(A)}(\rho_x, \rho_y, L) = < \varphi_1(x+\rho_x, y+\rho_y, L)\, \varphi_1^*(x, y, L) > = 2\pi\, p^2 L \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} d\omega$$

$$\cdot\, W_N\left[k_x, k_y, \frac{\omega}{V_* \cos\theta} - k_x\, tg\theta, \omega\right] \exp(i\, k_x\, \rho_x + i\, k_y\, \rho_y) \qquad (13)$$

the angle brackets indicate an ensemble average, the $*$ denotes a complex conjugate, L is a distance travelling by wave in magnetized plasma, $W_N(\mathbf{k},\omega)$ is arbitrary spectral function of plasma density fluctuations, ρ_x and ρ_y distances between observation points in the *XY* plane. Knowledge of the variance of the phase fluctuation allows estimating attenuation of the amplitude of an incident wave in turbulent plasma caused by energy transformation from the mean field to the scattered one using the well-known formula [21]: $< E > = E_0 \exp(- < \varphi_1^2 > / 2)$. Attenuation of the mean field in magnetized plasma is connected with transmission energy of this field into fluctuating one.
In contactless diagnostics of the nonstationary plasma the most important is the temporal spectrum of scattered waves. The variance of an instant frequency $< \omega_1^2 >$ determines the broadening of the temporal power spectrum easily measuring by experiment [9]. It can be obtained from Equation (13) multiplying integrand on the factor ω^2. Violation of coherence of a scattered field in medium with large-scale irregularities connecting with the phase fluctuations allows us to suppose that $< \omega_1^2 >$ keeps the sense in the presence of diffraction too. Curvature of a constant phase surface in turbulent plasma is characterized by fluctuations of the unit vector $\mathbf{s}$ perpendicular to the wave front: $< s_{1x}^2 > = < (\partial\varphi_1 / \partial\, \mathrm{x})^2 > / k_0^2$. Both statistical characteristics $< s_{1x}^2 >$ and $< s_{1y}^2 >$ determine the angle-of-arrival of scattered waves in the *XY* plane.

The obtained statistical characteristics of slow low-frequency Alfven waves in the *E*-region of ionosphere are valid for arbitrary correlation function of the density fluctuations of the neutral components taking into account: anisotropy factor of irregularities, the angle of inclination of prolate irregularities with respect to the external magnetic field, the angle between wave vector of an incident wave and the external magnetic field, regular velocity and characteristic spatial-temporal scales of the density fluctuations characterizing turbulent plasma flow. The condition offreezing-in of the geomagnetic field $\mathbf{H_0}$ is not fulfilled.

The most important problem of waves propagation in a nonstationary medium is energy exchange between wave and medium. Specific features arise at propagation of the Alfven waves in plasma flow with chaotically varing parameters. The solution of this problem for Alfven wave is based on the calculation of the mean energy flux density (MEFD) $\mathbf{S} = \eta\, E^2\, \mathbf{V_{gr}}$ [8,18]. Growth and decrease of the energy flow in the turbulent plasma means the energy transfer from medium to the wave and vice versa. Neglecting dissipation processes in the ray (-optics) approximation amplitude E satisfies the transport equation [20,10]:

$$\frac{\partial}{\partial t}(\eta E^2) + div(\mathbf{V_{gr}} \cdot \eta E^2) = -\frac{\partial \varepsilon}{\partial t} E^2 , \tag{14}$$

where: $\eta = \frac{1}{\omega}\frac{\partial}{\partial \omega}(\omega^2 \varepsilon_{xx})$ is the coefficient between the energy density and E^2 (20), $\varepsilon_{xx} = \frac{c^2}{V_a^2}\frac{(\omega - \mathbf{k}\,\mathbf{V_0})}{\omega^2}$ is the component of dielectric permittivity obtaining for a moving plasma applying method introducing in (18). For low-frequency Alfven waves we obtain [10]:

$$\eta = \frac{c^2}{V_a^2}\frac{2}{1 \pm \tilde{g}} \ , \quad \frac{\partial \varepsilon}{\partial t} = \frac{c^2}{V_{a0}^2 \, (1 \pm \tilde{g})^2}\frac{1}{N_0}\frac{\partial N_1}{\partial t} \ . \tag{15}$$

where $\tilde{g} = V_0 / V_{a0}$.

Using Equation (15) the MEFDs of scattered Alfven waves in the turbulent plasma flow are

$$< S_z >= 2E_0^2 \frac{c^2}{V_{a0}^2}\frac{V_* \cos\theta}{(1 \pm \tilde{g})^2}\left(1 \mp \frac{V_*^2}{V_{a0}^2}\frac{<\omega_1^2>}{\omega_0^2}\right) , \quad < S_x >= 2E_0^2 \frac{c^2}{V_{a0}^2}\frac{V_* \sin\theta}{1 \pm \tilde{g}}\left(1 \mp \frac{V_*^2}{V_{a0}^2}\frac{1}{1 \pm \tilde{g}}\frac{<\omega_1^2>}{\omega_0^2}\right), \tag{16}$$

From these formula follow that growth or decrease of the MEFD of the FAW and SAW in the turbulent plasma flow substantially depend on: the group velocity of this wave V_*, the degree of ionization of the ionosphere η and frequency of an incident wave frequency ω_0. In the direction of Z axis at small velocities of turbulent plasma flow $\tilde{g} < 1$ (i.e. $V_0 < V_{a0}$), fluctuations of neutral particles density lead to decrease of the MEFD for FAW and for SAW, vice versa, to the increase of the MEFD. This is a consequence of the fact that that energy of slow wave is negative and $\eta < 0$, hence if energy of slow wave is decreased, amplitude and intensity of this wave are grow (10). Increasing regular velocity of plasma flow $\tilde{g} > 1$ (i.e. $V_0 > V_{a0}$), character of the parametric energy exchange between both Alfven waves with nonstationary plasma is the same, however the influence of density fluctuations increases due to factor $\tilde{g}^2$ before statistical parameter $<\omega_1^2>/\omega_0^2$ describing the broadening of the temporal spectrum. It is easily to show that at $\tilde{g} < 1$ for both Alfven waves $< S_x >= tg\,\theta < S_x >$. This is a consequence of anisotropy of the task connecting with the regular velocity of plasma flow. Increasing V_0, $\tilde{g} > 1$ we obtain $< S_x > \approx \tilde{g}\; tg\,\theta < S_x >$.

Scattered field is connected with the log-amplitude by the relation $\chi = \ln(E / E_0)$ [21,22]. Using the transport equation (14) solution of the stochastic differential equation for the amplitude of a scattered field has the following form

$$\chi_1(\mathbf{r},t) = -\frac{1}{4(1 \pm \tilde{g})\, N_0 V_* \cos\theta}\int_0^L dz' \frac{\partial}{\partial t'} N_1(x', y, z', t') , \tag{17}$$

where integral should be taken along the characteristics: $x' = x - (z - z')\, tg\theta$, $t' = t - (z - z') / V_* \cos\theta$. Comparing variances of the amplitude and frequency fluctuations for both Alfven waves we obtain:

$$<\chi_1^2> = \frac{<\omega_1^2>}{4\omega_0^2}\frac{V_*^2}{V_{a0}^2} . \tag{18}$$

If $\tilde{g} < 1$ we get $< \chi_1^2 > = < \omega_1^2 > / 4\,\omega_0^2$, if $\tilde{g} > 1$, we have $< \chi_1^2 > = \tilde{g}^2 < \omega_1^2 > / 4\,\omega_0^2$. Increasing regular velocity of plasma flow, amplitude fluctuations for both Alfven waves are increased and hence intensity of scattered wave growth in proportion to distance travelling by waves in plasma $< E^2 > \sim E_0^2 < \chi_1^2 >$.

It should be noted that if the conditions: $|V_0 - V_{a\,0}| << V_{a\,0}$ for waves is fulfilled, the wavelength becomes very small and application of the geometrical optics approximation to the slow large-scale MHD waves is violated. All above derived formulae are valid for the angles θ not close neither zero nor $\pi/2$, because at small angle θ phase velocities of the Alfven and magnetoacoustic waves approximately coincide and strong linear interaction takes place [20] which we did not take into account. The second-order statistical moments calculating in the ray (-optics) approximation not include diffraction effects and impose the restriction on distance $L / k_0\, l^2 << 1$ [21,22].

NUMERICAL CALCULATIONS

Observations of ionospheric irregularities detected by radio wave sounding of the lower *E*-region (altitudes near 100 km) have shown [23] that the speeds and horizontal spatial scales of the dominant irregularities ranged between 30 and 160 $\mathrm{m \cdot s^{-1}}$ and between 10 and 75 km, respectively; with the corresponding average values being near 80 $\mathrm{m \cdot s^{-1}}$ and 30 km. The mean drift speed in the *E*- region of ionosphere is of an order 100-150 $\mathrm{m \cdot s^{-1}}$ depending on geomagnetic activity. Below 110 km drift velocity coincides with the wind speed; above 130 km ionized component drifts towards the direction of an external magnetic field. In the plane perpendicular to a geomagnetic lines of force drift speed by an order of magnitude is less than a speed of the wind.

Large-scale anisotropic irregularities have been observed in the *E*-region of ionosphere. Horizontal spatial scale of these irregularities is about 150-200 km. They generated due to wavy movements of an internal waves. Inhomogeneous structure of the ionosphere is investigated [24] using the space diversity techniques. Observations have shown that anisotropic coefficient of irregularities at $\chi < 5$ is not connected with the geomagnetic field, but substantial elongation $\chi \geq 10$ is defined by it. Velocities of irregularities movement is in the range of $40 \div 160\ \mathrm{m \cdot s^{-1}}$; the most probable drift speed is $\sim 100\ \mathrm{m \cdot s^{-1}}$ that is an agreement with other experimental data. The variance of concentration $\sigma_N^2 = < N_1^2 > / N_0^2$ was measured using pulse and radio-astronomical methods. Observations of the *E*-region have shown that characteristic linear scale of irregularities is about 1-2 km and $\sigma_N^2 \sim 10^{-4}$.

Analytical and numerical calculations will be carried out for anisotropic Gaussian correlation function of density of the neutral components having in the principle plane following form [25]:

$$W_N(\mathbf{k}, \omega) = \sigma_N^2\, \frac{l_\perp^2\, l_\parallel\, T}{16\pi^2} \exp\left(- p_1 \frac{k_x^2\, l_\parallel^2}{4} - \frac{k_y^2\, l_\perp^2}{4} - p_2 \frac{k_z^2\, l_\parallel^2}{4} - p_3\, k_x\, k_z\, l_\parallel^2 - \frac{\omega^2 T^2}{4} \right), \tag{19}$$

where:. $p_1 = (\sin^2\gamma_0 + \chi^2 \cos^2\gamma_0)^{-1} \left[1 + (1-\chi^2)^2 \sin^2\gamma_0 \cos^2\gamma_0 / \chi^2 \right]$, $p_2 = (\sin^2\gamma_0 + \chi^2\cos^2\gamma_0) / \chi^2$, $p_3 = (1-\chi^2)\sin\gamma_0 \cos\gamma_0 / 2\chi^2$, $\sigma_N^2 = < N_1^2 > / N_0^2$ is variance of neutral particles density fluctuations. This function contains anisotropy factor of irregularities $\chi = l_\parallel / l_\perp$ (ratio of longitudinal and transverse linear scales of plasma irregularities) and inclination angle γ_0 of prolate irregularities with respect to the external magnetic field.

Substituting Eq. (19) into Eq. (13) for the fast Alfven wave in the polar coordinate system we obtain:

$$V_{\varphi}^{(+)}(X,Y,L,\varphi) = \frac{0.39}{4\sqrt{\pi}} \frac{(\omega_0 T)^2}{g_0 \chi^2 \cos^2\theta} M_A^2 \frac{1}{g_5^2} \left[1 - \frac{2B_1^2}{g_5^2} + i\sqrt{\pi} \frac{B_1}{g_5} \exp\left(\frac{B_1^2}{g_5^2} \right) \right], \qquad (20)$$

where: $g_0 = \left[1 + \frac{p_2}{\cos^2\theta} M_A^2 \right]^{1/2}$, $g_2 = p_1 + p_2\, tg^2\theta - 4\, p_3\, tg\theta$, $g_3 = \left(\frac{1}{2} p_2\, tg\theta - p_3 \right)^2$, $M_A = l_{\parallel} / V_* T$,

$g_4 = \left(g_2 - \frac{2a}{\cos^2\theta} \right)^{1/2}$ $(g_4^2 > 0)$, $a = \frac{2g_3}{g_0^2} M_A^2$, $g_5 = \left(g_4^2 \cos^2\varphi + \frac{\sin^2\varphi}{\chi^2} \right)^{1/2}$, $B_1 = X\cos\varphi + Y\sin\varphi$.

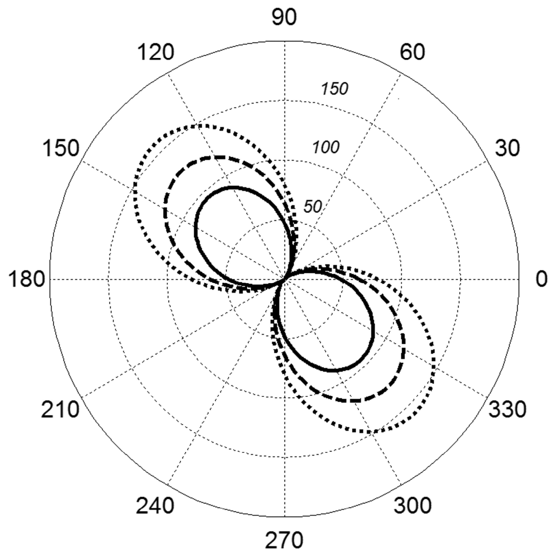

Fig 1: depicts phase portrait of fast Alfven wave in the turbulent plasma flow $V_0 / V_{a0} = 0.6$ and different non-dimensional spatial-temporal parameter M_a characterizing nonstationary plasma; Ma =15 – solid line, Ma = 20 - dashed line, Ma = 25 - dotted line.

Numerical calculations for the fast Alfven wave were carried out for the following plasma parameters: $\gamma_0 = 5^0$, $\chi = 5$, $X = Y = 0.1$, $V_0 / V_{a0} = 0.6$ (solid line-Ma=15, dashed line- Ma = 20, dotted line- Ma = 25). Analysis show that the spatial-temporal fluctuations of plasma flow impose substantial influence on the phase portrait and directional fluctuations of fast Alfven wave in the turbulent plasma stream. Numerical calculations for slow Alfven wave were carried out for the following plasma parameters: $\gamma_0 = 15^0$, $\chi = 5$, $X = Y = 0.08$, $V_0 / V_{a0} = 0.6$ and fixed parameter Ma = 10 (solid line- $\chi = 7$, dashed line- $\chi = 6$, dotted line- $\chi = 5$, dashed dotted line $\chi = 4$). Analysis show that the spatial-temporal fluctuations of plasma flow impose substantial influence on the phase portrait and directional fluctuations of fast Alfven wave in the turbulent plasma stream. Numerical calculations for slow Alfven wave were carried out for the following plasma parameters: $\gamma_0 = 15^0$, $\chi = 5$, $X = Y = 0.08$, $V_0 / V_{a0} = 0.6$ and fixed parameter Ma = 10 (solid line- $\chi = 7$, dashed line- $\chi = 6$, dotted line- $\chi = 5$, dashed dotted line $\chi = 4$). Temporal pulsations of plasma density fluctuations lead to the broadening of the temporal spectrum. Fig. 3 depicts the dependence of the

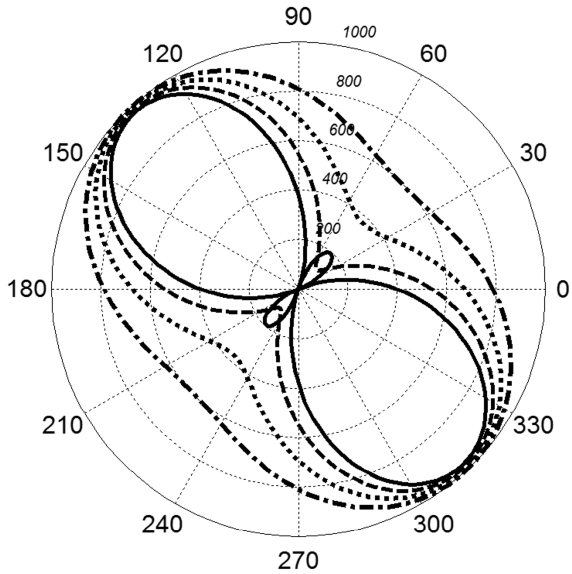

Fig 2: depicts phase portrait of slow Alfven wave in the turbulent plasma flow $V_0 / V_{a0} = 0.6$ and different non-dimensional spatial-temporal parameter M_a characterizing nonstationary plasma; Ma =15 – solid line, Ma = 20 - dashed line, Ma = 25 - dotted line.

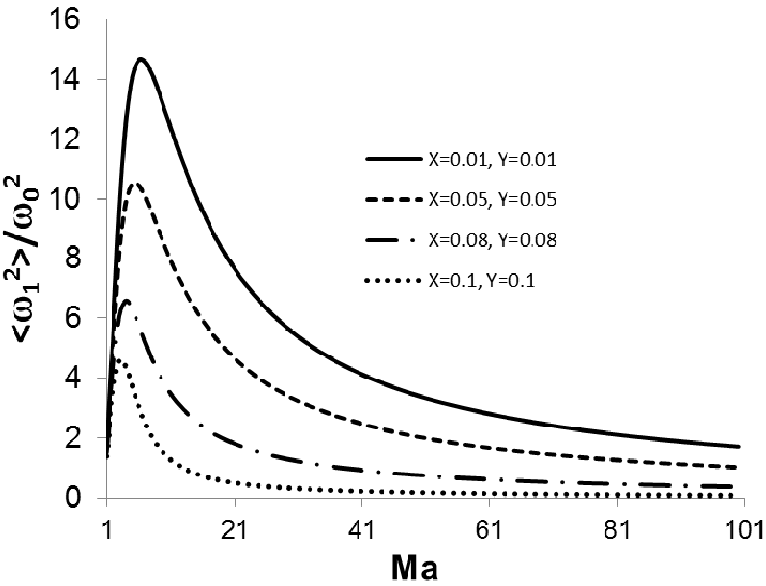

Fig 3: illustrates broadening of the temporal power spectrum of scattered Alfven wave in the turbulent plasma flow versus non-dimensional spatial-temporal parameter M_a characterizing nonstationary plasma for the anisotropic Gaussian spectrum and different locations of the observation points.

normalized variance of the frequency fluctuations characterizing broadening of the temporal power spectrum versus non-dimensional parameter $M_A = l_{\parallel} / V_* T$ containing all characteristic spatial-temporal scales of the turbulent plasma flow at different location of the observation points. Using experimental data: $\theta = 30^0$, $\chi = 3$, $\gamma_0 = 15^0$, $V_{a0} = 50\ \text{m} \cdot \text{s}^{-1}$, $V_0 = 100\ \text{m} \cdot \text{s}^{-1}$, $l_{\parallel} = 30\,\text{km}$. Numerical calculations show that at: $X = Y = 0.01$ (observation points are spaced apart at distances $\rho_x = \rho_y = 300\,\text{m}$), maximum of the temporal spectrum of scattered Alfven wave in the turbulent plasma flow is at the frequency $\nu_{\max} = 6\,\text{mHz}$ and the frequency band of a half width of the temporal spectrum is equal $\Delta\nu = 20\,\text{mHz}$. Increasing distance between observation points, $X = Y = 0.08$ ($\rho_x = \rho_y = 2.4\,\text{km}$), maximum and half width of the temporal spectrum are equal to: $\nu_{\max} = 4\,\text{mHz}$, $\Delta\nu = 10\,\text{mHz}$, respectively. At $X = Y = 0.1$ ($\rho_x = \rho_y = 3\,\text{km}$), $\nu_{\max} = 3\,\text{mHz}$, $\Delta\nu = 6\,\text{mHz}$. Considered above problems have direct relation to the problem of generation, registration and propagation of VLF radiation which are devoted great attention in geophysical applications. The obtained results could be application at investigation of solar and galactic plasma.

CONCLUSIONS

It was established that in the weakly ionized E region of ionosphere exist: magneto-acoustic wave with frequency $\omega_M = V_a k_x$, Alfven wave with frequencies ω_{A*} and $\omega_A = V_a k_z$, slow cyclotron wave of ions Ω_i and helicons with frequency ω_h.

The peculiarities of large-scale slow MHD waves in weakly ionized ionosphere with randomly varying spatial-temporal parameters are considered on the bases of dispersion equation. It was shown that slow Alfven waves propagating with the phase velocity of the order of $20\,\text{m} \cdot \text{s}^{-1}$ transverse to the external magnetic field lead to the substantial distortion of the geomagnetic field within the range of 15-50 nT. The obtained results are in agreement with experimental observations confirming the existence of planetary waves with velocities of 20-100 $\text{m} \cdot \text{s}^{-1}$ in the E -layer of ionosphere in any season of year with wavenumber of 2-10 m^{-1}. Large-scale disturbances (with the velocity of 1-2 $\text{km} \cdot \text{s}^{-1}$) can be identified with magneto-acoustic waves in the neutral component of the ionosphere, and the planetary waves with the velocity of the motion 20-100 $\text{m} \cdot \text{s}^{-1}$ can be identified with slow Alfven waves; fast and slow Alfven waves are circularly polarized taking into account dispersion (Hall's effect).

Second order statistical moments: correlation functions of the phase fluctuations of scattered Alfven wave, broadening of the temporal power spectrum and the mean square of log-amplitude fluctuations describing angle-of-arrival of scattered fast and slow Alfven waves in a turbulent plasma flow have been obtained in the ray (optics) approximation for arbitrary correlation function of the density fluctuations of the neutral component taking into account: anisotropy factor of irregularities, the angle of inclination of prolate irregularities with respect to the external magnetic field, the angle between wave vector of an incident wave and the external magnetic field, regular velocity and characteristic spatial-temporal scales of the density fluctuations characterizing turbulent plasma flow. Knowledge of the variance of the phase fluctuation allows estimating attenuation of the amplitude of an incident wave in turbulent plasma caused by energy transformation from the mean field to the scattered one.

Energy exchange between Alfven wave and turbulent plasma flow is investigated analytically and numerically using experimental data. Statistical parameters of scattered Alfven waves substantially depend on the ration of Alfven velocity and macroscopic velocity of a plasma flow. Characteristic frequencies of the temporal pulsations of plasma irregularities leading to the broadening of the temporal spectrum are calculated first time.

Phase portraits are constructed for "fast" and "slow" Alfven waves in the turbulent plasma flow using the experimental data. It was shown that the turbulent plasma parameters have a substantial influence on the directional fluctuations of scattered MHD waves.

REFERENCES

[1]. AG Khantadze; GV Jandieri; A. Ishimaru; ZhM Diasamidze; Annales Geophysicae, **2010**, 28, 1387.
[2]. GV Jandieri; A. Ishimaru; VG Gavrilenko; AA Surmava; AI Gvelesiani; The Open Atmospheric Science Journal, **2011**, 5, 33.
[3]. Y Kamide; W Baumjohann; Magnetosphere-ionosphere coupling, Springer, Berlin. **1993**
[4]. VM Sorokin; GV Fedorovich; Physics of slow MHD waves in the ionospheric plasma, Nauka, Moscow, 1982 (in Russian).
[5]. DJ Cavalieri; RJ Deland; JF Poterna; RF Gavin; J. Atmos. Terr. Phys., **1974**, 36, 561.
[6]. ES Kazimirovskii; VD Kokourov; Motions in the ionosphere. Nauka: Novosibirsk, Russia (in Russian), **1979**.
[7]. LS Al'perovich; EA Ponomarev; GV Fedorovich; Izv. Phys. Solid Earth **1985**.
[8]. AI Akhiezer; IA Akhiezer; RV Polovin; AG Sitenko; KN Stepanov; Electrodynamics of Plasma, Nauka, Moscow (in Russian), **1974**.
[9]. YuA Kravtsov; LA Ostrovsky; NS Stepanov; Proc. IEEE, 1974, 62, **1492**.
[10]. VG Gavrilenko; GV Jandieri; AA Semerikov; Plasma Physics Report, **1985**, 11, 1193.
[11]. MC Kelley; The Earth's ionosphere, Academic, San Diego, Calif. **1989**,
[12]. LS Alperovich; EN Fedorov; Hydromagnetic waves in the magnetosphere and the ionosphere, Springer, **2007**, 425 p.
[13]. AS Monin; AM Obukhov; Izv.,Acad. Nauk Geol., **1958**, 11, 1360.
[14]. AG Khantadze; GD Aburjania; GV Jandieri; Plasma Physics Report, **2004**, 30, 83.
[15]. AG Khantadze; ZS Sharadze; In: Wave Disturbances in Atmosphere. Alma Ata: Nauka, **1980**, 143.
[16]. AG Khantadze; ZS Sharadze; In: Research of Dynamical Process in Upper Atmosphere. Nauka, Moscow, **1988**, 110.
[17]. IS Dmitrenko; VA Mazur; Planetary and Space Science, **1985**, 33, 471.
[18]. AF Aleksandrov; LS Bogdankevich; AA Rukhadze; *Electrodynamics of Plasma*, Moscow: Higher Educational Institution (in Russian), **1988**.
[19]. BB Kadomtsev; Collective phenomena in plasma, Pergamon Press, New York, **1982**.
[20]. YuA Kravtsov; YuI Orlov Geometrical optics of inhomogeneous media, Nauka, Moscow (in Russian), **1980**,.
[21]. SM Rytov; YuA Kravtsov; VI Tatarskii; Principles of Statistical Radiophysics. vol.4. Waves Propagation Through Random Media. Springer, Berlin, New York, **1989**.
[22]. A. Ishimaru; Wave Propagation and Scattering in Random Media, Vol. 2, Multiple Scattering, Turbulence, Rough Surfaces and Remote Sensing, IEEE Press, Piscataway, New Jersey, USA, **1997**.
[23]. RA Vincent; Journal of Atmospheric and Solar Terrestrial Physics, **1972**, 34, 1881.
[24]. YuL Kokurin; In Proceedings Drifts and irregularities in the ionosphere, **1959**, 1, 62.
[25].GV Jandieri; A. Ishimaru; VG Jandieri; AG Khantadze; ZhM Diasamidze; Progress In Electromagnetics Research, **2007**, 70, 307.