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Modification to the matrix method for the direct and inverse problems of seismology

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ABSTRACT

The modification of the matrix method of construction of wavefield on the free surface of an anisotropic medium is presented. The earthquake source represented by a randomly oriented force or a seismic moment tensor is placed on an arbitrary boundary of a layered anisotropic medium. The theory of the matrix propagator in a homogeneous anisotropic medium by introducing a "wave propagator" is presented. It is shown that for anisotropic layered medium the matrix propagator can be represented by a "wave propagator" in each layer. The matrix propagator $P(z,z_0=0)$ *acts on the free surface of the layered medium and generates stress-displacement vector at depth z. The displacement field on the free surface of an anisotropic medium is obtained from the received system of equations considering the radiation condition and that the free surface is stressless. The approbation of the modification of the matrix method for isotropic and anisotropic media with TI symmetry is done. A comparative analysis of our results with the synthetic seismic records obtained by other methods and published in foreign papers is executed.*

Keywords: matrix method, seismic tensor, synthetic seismograms

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INTRODUCTION

The main data sources in seismology are the seismic records of natural or man-made events that are received on the Earth surface. The task of modern seismic analysis is to obtain the maximum possible information about the nature of wave-fields propagation. Solving these problems involves the study of seismic regions of Ukraine and interpretation of wave fields in order to determine the earthquake focal mechanisms.

In recent years one of the most important methods is the development of approaches for constructing the theoretical seismograms, which allow the study of the structure of the medium and determination of the earthquake source parameters. The effects on the wave field and seismic waves' propagation in the Earth's interior should be considered when calculating these seismograms. Thus, the displacement field, which is registered on the free surface of an inhomogeneous medium, depends on the model of the geological structure and the physical processes in the source.

In the 50's of 20th century Thomson and Haskell first proposed a method for constructing interference fields by simulation of elastic waves in layered isotropic half-space with planar boundaries [*Haskell N.A.*, 1953]. The matrix method was developed in the works [*Behrens E.*, 1967; *Buchen P.W*., 1996; *Cerveny V.*, 2001; *Chapman, C.H.*, 1974].

The stable algorithms of seismograms calculation for all angles of seismic waves propagation is obtained. The matrix method is generalized for low-frequency waves in inhomogeneous elastic concentric cylindrical and spherical layers surrounded by an elastic medium. The concept of the characteristic matrix determined by physical parameters of the environment is developed. The matrix method is used for wave propagation in elastic, liquid and thermoelastic media. In addition, it has been generalized for the study of other processes described by linear equations. The advantage of the matrix method is the ability to compactly write matrix expressions that are useful both in analytical studies and numerical calculations.

The matrix method and its modifications are used to simulate the seismic waves propagation in isotropic and anisotropic media. This method is quite comfortable and has several advantages over other approaches. Both advantages and disadvantages of the matrix method are well described in [*Helbig K., et al*., 2001); *Stephen R. A.*, 1981; *Thomson W.T.*, 1950].

Today in seismology much attention is given to mathematical modelling as one of the main tools for the analysis and interpretation of the wave fields.

MATERIALS AND METHODS

Theory of the modification to the matrix method

The problem of wave fields modelling, when the source is presented by seismic moment, has practical applications in seismology. Therefore, the development of methods for determining the displacement field on the free surface of an anisotropic inhomogeneous medium for sources of this type is an actual task and needs to be resolved.

In this paper the propagation of seismic waves in anisotropic inhomogeneous medium is modelled by system of homogeneous anisotropic layers, as shown in (Fig. 1). The each layer is characterized by the propagation velocity of P- and S-wave and density. At the boundaries between layers hard contact condition is met, except for the border, where the source of seismic waves is located.

The earthquake source is modelled by nine pairs of forces, which represented a seismic moment tensor. This description of the point source is sufficiently known and effective for simulation of seismic waves in layered half-space [*Haskell N.A.*, 1953].]. In general, the source is also assumed to be distributed over time, i.e. seismic moment $M_0(t)$ is a function of time. This means that the physical process in the source does not occur instantaneously, but within a certain time frame. It is known for our seismic events (Mw ~ 2-3) that the time during which occurred the event may be 0.1 – 0.7 seconds. The determination of the source time function is an important seismic problem. In this chapter the direct problem solution is shown, when a point source is located on an arbitrary boundary of layered anisotropic media.

We assume the usual linear relationship between stress τ_{ij} and strain e_{kl}

$$\tau_{ij} = c_{ijkl} \cdot e_{kl} = c_{ijkl} \frac{\partial u_l}{\partial x_k} \qquad (1)$$

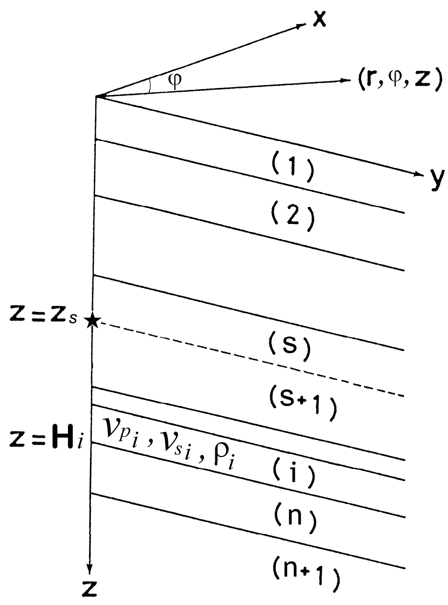

Fig. 1. Model vertically inhomogeneous medium

where $u=(u_x, u_y, u_z)^T$ is displacement vector.
The equation of motion for an elastic homogeneous anisotropic medium, in the absence of body forces is [*Fryer et al*, 1984]

$$\rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} \frac{\partial^2 u_l}{\partial x_i \partial x_k} \qquad (2)$$

where ρ is the uniform mass density, and c_{ijkl} are the elements of the uniform elastic coefficient tensor.

Taking the Fourier transform of (1) and (2), we obtain the matrix equation [*Fryer et al*, 1987]

$$\frac{\partial \vec{b}}{\partial z} = j\omega A(z)\vec{b}(z) \qquad (3)$$

where $\vec{b} = \begin{pmatrix} \vec{u} \\ \vec{\tau} \end{pmatrix}$ is the vector of displacements and scaled tractions, $\vec{\tau} = -\frac{1}{j\omega}(\tau_{xz}, \tau_{yz}, \tau_{zz})^T$. With the definition of b the system matrix A has the structure

$A = \begin{pmatrix} T & C \\ S & T^T \end{pmatrix}$; where *T*, *S* and *C* are 3×3 sub matrices, *C* and *S* are symmetric.

For any vertically stratified medium, the differential system (3) can be solved subject to specified boundary conditions to obtain the response vector b at any desired depth. If the response at depth z_0 is $b(z_0)$, the response at depth z is

$$\vec{b}(z) = P(z, z_0)\vec{b}(z_0) \qquad (4)$$

where $P(z, z_0)$ is the stress-displacement propagator.
To find this propagator, it is necessary to find the eigenvalues (vertical slownesses), the eigenvector matrix D, and its inverse D^{-1} [*Fryer et al*, 1984]:

$$P(z, z_1) = DQ(z, z_1)D^{-1}, \qquad (5)$$

where Q is the “wave” propagator [*Fryer et al*, 1984]:

$$Q(z,z_1)=\begin{pmatrix}E_u & 0\\ 0 & E_D\end{pmatrix} \qquad (6)$$

where $E_u = diag[e^{j\omega(z-z_1)q_p^u}, e^{j\omega(z-z_1)q_{s_1}^u}, e^{j\omega(z-z_1)q_{s_2}^u}]$, $E_D = diag[e^{j\omega(z-z_1)q_p^D}, e^{j\omega(z-z_1)q_{s_1}^D}, e^{j\omega(z-z_1)q_{s_2}^D}]$.

In the isotropic case the eigenvector matrix D known analytically, so the construction of the propagator is straightforward. In the anisotropic case, analytic solutions have been found only for simple symmetries so in general, solutions will be found numerically.

The layered anisotropic medium, which consists of *n* homogeneous anisotropic layers on an anisotropic halfspace (*n* +1) (Fig. 1), is considered. The matrix propagator (4*) can be represented by a “wave propagator” in each layer for anisotropic layered medium. The source in the form of a jump in the displacement-stress $\vec{F}=\vec{b}_{s+1}-\vec{b}_s$ is placed on the s-boundary (Fig. 1); it is easy to write the following matrix equation, using (13-14):

$$\vec{b}_{n+1}=P_{n,s}\vec{b}_{s+1}\Big|_{z=z_s}, \quad v_{n+1}=D_{n+1}^{-1}D_nQ_nD_n^{-1}\cdots D_{s+1}Q_{s+1}D_{s+1}^{-1}\cdot\vec{b}_{s+1}\Big|_{z=z_s},$$

$$\vec{b}_s\Big|_{z=z_s}=P_{s,s-1}P_{s-1,s-2}\cdots P_{2,1}P_{1,0}\cdot\vec{b}_0=D_sQ_sD_s^{-1}\cdots D_1Q_1D_1^{-1}\cdot\vec{b}_0,$$

$$\begin{aligned}v_{n+1}&=D_nQ_nD_n^{-1}\cdots D_{s+1}Q_{s+1}D_{s+1}^{-1}\cdot(\vec{b}_s+\vec{F})=G^{n+1,s+1}\cdot(G_{s,1}\vec{b}_0+\vec{F})=\\ &G^{n+1,s+1}G_{s,,1}\vec{b}_0+G^{n+1,s+1}\cdot\vec{F}=G\vec{b}_0+G^{n+1,s+1}\cdot\vec{F}\end{aligned},$$

where

$$G=D_{n+1}^{-1}D_nQ_nD_n^{-1}\cdots D_{s+1}Q_{s+1}D_{s+1}^{-1}\cdots D_2^{-1}D_1Q_1D_1^{-1}$$

- characteristic matrix of a layered anisotropic medium.

$$\vec{v}_{n+1}=G\vec{b}_0+G\cdot G_{s,1}^{-1}\cdot\vec{F}=G(\vec{b}_0+G_{s,1}^{-1}\cdot\vec{F})=G(\vec{b}_0+\tilde{\vec{F}}), \qquad (7)$$

where $\tilde{\vec{F}}=G_{s,1}^{-1}\cdot\vec{F}$, $G=G^{n+1,s+1}\cdot G_{s,1}$.

Using (7) and the radiation condition (with a halfspace (*n*+1) the waves are not returned), and also the fact that the tension on the free surface equals to zero, we obtain a system of equations:

$$\begin{pmatrix}0\\0\\0\\v_D^P\\v_D^{S_1}\\v_D^{S_2}\end{pmatrix}=\begin{pmatrix}G_{11}&G_{12}&G_{13}&G_{14}&G_{15}&G_{16}\\G_{21}&G_{22}&G_{23}&G_{24}&G_{25}&G_{26}\\G_{31}&G_{32}&G_{33}&G_{34}&G_{35}&G_{36}\\G_{41}&G_{42}&G_{43}&G_{44}&G_{45}&G_{46}\\G_{51}&G_{52}&G_{53}&G_{54}&G_{55}&G_{56}\\G_{61}&G_{62}&G_{63}&G_{64}&G_{65}&G_{66}\end{pmatrix}\begin{pmatrix}u_x^{(0)}+\tilde{F}_1\\u_y^{(0)}+\tilde{F}_2\\u_z^{(0)}+\tilde{F}_3\\\tilde{F}_4\\\tilde{F}_5\\\tilde{F}_6\end{pmatrix}.$$

Using only the homogeneous equations is sufficient to get the displacement field on a free surface:

$$\begin{cases}G_{11}u_x^{(0)}+G_{12}u_y^{(0)}+G_{13}u_z^{(0)}=-(G_{11}\tilde{F}_1+G_{12}\tilde{F}_2+G_{13}\tilde{F}_3+G_{14}\tilde{F}_4+G_{15}\tilde{F}_5+G_{16}\tilde{F}_6)\\G_{21}u_x^{(0)}+G_{22}u_y^{(0)}+G_{23}u_z^{(0)}=-(G_{21}\tilde{F}_1+G_{22}\tilde{F}_2+G_{23}\tilde{F}_3+G_{24}\tilde{F}_4+G_{25}\tilde{F}_5+G_{26}\tilde{F}_6)\\G_{31}u_x^{(0)}+G_{32}u_y^{(0)}+G_{33}u_z^{(0)}=-(G_{31}\tilde{F}_1+G_{32}\tilde{F}_2+G_{33}\tilde{F}_3+G_{34}\tilde{F}_4+G_{35}\tilde{F}_5+G_{36}\tilde{F}_6)\end{cases}.$$

The stress-displacement discontinuity is determined via the seismic in matrix form [*Fryer G.J. et al*, 1984]:

$$\vec{F} = \begin{pmatrix} -c_{55}^{-1}M_{xz} \\ -c_{44}^{-1}M_{yz} \\ -c_{33}^{-1}M_{zz} \\ p_x(M_{xx} - c_{13}c_{33}^{-1}M_{zz}) + p_yM_{xy} \\ p_xM_{yx} + p_y(M_{yy} - c_{23}c_{33}^{-1}M_{zz}) \\ p_x(M_{zx} - M_{xz}) + p_y(M_{zy} - M_{yz}) \end{pmatrix} \delta(z - z_z)$$

where M_{xx}, M_{yy}, M_{zz}, M_{xz}, M_{yz}, M_{yx}, M_{xy}, M_{zy}, M_{zx} – components of the seismic moment tensor, and c_{13}, c_{23}, c_{33}, c_{44}, c_{55} – components of the stiffness matrix.

As a result, the displacement field of the free surface of an anisotropic medium is in the spectral domain as:

$$\vec{u} = \begin{pmatrix} u_x^0 \\ u_y^0 \\ u_z^0 \end{pmatrix} = (G^{13})^{-1} \cdot \vec{y}, \qquad (8)$$

where $G^{13} = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}$, $\vec{y} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$,

$$a = -(G_{11}\tilde{F}_1 + G_{12}\tilde{F}_2 + G_{13}\tilde{F}_3 + G_{14}\tilde{F}_4 + G_{15}\tilde{F}_5 + G_{16}\tilde{F}_6),$$

$$b = -(G_{21}\tilde{F}_1 + G_{22}\tilde{F}_2 + G_{23}\tilde{F}_3 + G_{24}\tilde{F}_4 + G_{25}\tilde{F}_5 + G_{26}\tilde{F}_6),$$

$$c = -(G_{31}\tilde{F}_1 + G_{32}\tilde{F}_2 + G_{33}\tilde{F}_3 + G_{34}\tilde{F}_4 + G_{35}\tilde{F}_5 + G_{36}\tilde{F}_6).$$

Using (8) and three-dimensional Fourier transform, we obtain a direct problem solution for the displacement field of the free surface of an anisotropic medium in the time domain as:

$$\vec{u}(x, y, z_R, t) = \frac{1}{8\pi^3} \iiint_{-\infty} \omega^2 \vec{u}(p_x, p_y, z_R, \omega) e^{j\omega(t - p_x x - p_y y)} dp_x dp_y d\omega,$$

where z_R – epicentral distance, p_x, p_y- horizontal slowness.

Analytical and numerical approaches of determining of the source parameters in case when source is presented by randomly oriented force

The seismic source may be described by a model of equivalent forces that correspond to linear wave equations. These sources can be analysed in a unified and consistent way by using the concept of the seismic moment tensor, which encapsulates the equivalent forces model of a generalised point source. The full set of force couples that comprise the moment tensor may be summed in a variety of different combinations to produce a wide range of seismic source models. This aspect illustrates the great utility of the moment tensor.

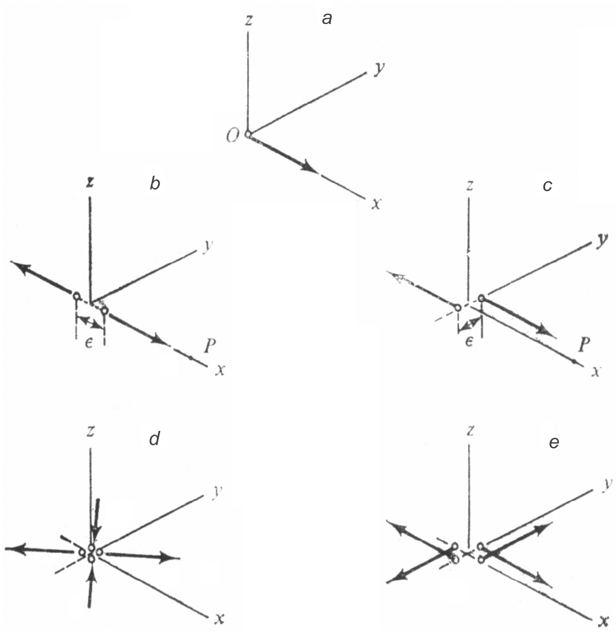

Fig. 2. Various types of forces in the point source.

a – a single force; b - a pair of forces equal in magnitude and opposite in direction relative to the axis z; c - a pair of forces equal in magnitude and opposite in direction relative to the axis z; d - two pairs of forces, which are equal in magnitude and act along axes perpendicular to each other; e - two pairs of forces equal in magnitude and opposite in direction relative to the axis z.

A seismic source may be described by a model of equivalent forces, corresponding to linear wave equations where non-linear effects in the near-source region are neglected. Equivalent forces are defined as those forces producing displacements at a given point that are identical to the displacements produced by the actual forces of the physical process and acting at the source. The concept of equivalent forces is a useful one, because these forces can be correlated with physical source models.

As a result, the solution of the direct problem of seismology, the synthetic seismograms for media with different types of anisotropy (transverse-isotropic symmetry orthorhombic and monoclinic anisotropy) are calculated. The stress-displacement discontinuity is determined by a randomly oriented force as [32]:

$$\vec{F} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{j\omega} f_x \\ \frac{1}{j\omega} f_y \\ \frac{1}{j\omega} f_z \end{pmatrix} \delta(z - z_0) \qquad (9)$$

where j – imaginary unit,

ω – angular frequency,

f_x, f_y, f_z – components of a randomly oriented force.

Using the obtained solution of the direct problem, we can solve the matrix equation (8) with respect to the stress-displacement discontinuity (9), where $\vec{u}^{(0)} = (u_x^{(0)}, u_y^{(0)}, u_z^{(0)})^T$ is a vector of displacements on the free surface in the spectral domaine.

The matrix equation is written in terms of the stress-displacement discontinuity as:

$$\vec{F} = G^{13} \cdot \vec{u}^{(0)} (G^{16})^{-1} \cdot G_{s,1}, \tag{10}$$

where ,

$$G^{16} = \begin{pmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & G_{16} \\ G_{21} & G_{22} & G_{23} & G_{24} & G_{25} & G_{26} \\ G_{31} & G_{32} & G_{33} & G_{34} & G_{35} & G_{36} \end{pmatrix}$$

Using (10) and three-dimensional Fourier transform, we obtain the analytical expression for determining the source time function (STF(t)) in the time domain.

If the case, when the seismic source is presented by randomly oriented force (9), the inverse problem solution is the source time function. To obtain the inverse problem solution, we need the three components of seismograms (u_x, u_y, u_z) and parameters of the medium (velocity model or stiffness matrix of the medium). If the real seismograms are used, the best results are obtained for the filtered real records up to 5 Hz in range.

Approbation of the new approach to determining the source time function

To test the proposed theory three examples are considered. The synthetic seismograms are calculated for the anisotropic medium. The seismic source is modelled by a randomly oriented force which is given as the source time function (STF(t)).

In the test case, all parameters of the anisotropic medium (11) and the coordinates of a seismic wave source are known.

For each of the following simulations the medium is modelled by anisotropic half-space with monoclinic symmetry. The seismic wave source is located at the depth 5 km; an epicentral distance to the receiver is 1 km.

The stiffness matrix of the medium is defined as:

$$c = \begin{pmatrix} 95.46 & 28.93 & 4.03 & 0 & 0 & 44.67 \\ 28.93 & 25.91 & 4.56 & 0 & 0 & 15.56 \\ 4.03 & 4.56 & 16.34 & 0 & 0 & 0.56 \\ 0 & 0 & 0 & 4.44 & -1.78 & 0 \\ 0 & 0 & 0 & -1.78 & 6.54 & 0 \\ 44.67 & 15.56 & 0.54 & 0 & 0 & 32.98 \end{pmatrix} \cdot 10^9 . \tag{11}$$

1. The source time function is preset as Wavelet function:

$$STF(t) = A \cdot \left(1 - 2\pi^2 f^2 t^2\right) e^{-(\pi f t)^2}, \tag{12}$$

where $A = 10^8$, $f = 5\text{Hz}$.

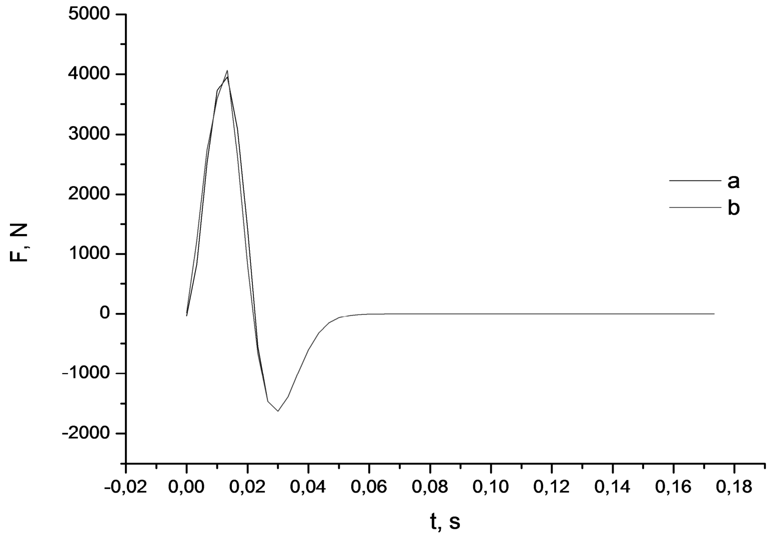

Fig. 3. Comparison of graphs STF (t) - Wavelet function: a – calculated in the inverse problem, b – constructed in the direct problem.

After analyzing the results, we can conclude that the inverse problem solution is fairly accurate. The correlation coefficient between the source time function (12) and the obtained inverse problem solution is equal to 0.9901.

2. The source time function is preset in spectral domain as:

$$STF(w) = \frac{A}{1 + \frac{w^2}{w_0^2}} \ , \qquad (13)$$

where $A = -10^8$, $\omega_0 = 5$Hz.

For the second example, when a randomly oriented force in the spectral domain is given as (13), the correlation coefficient between the function (2.5) and the obtained solution of the inverse problem is equal to 0.9928.

3. The source time function is a fading sinusoid. In the spectral domain function is:

$$STF(w) = A \cdot \left(\frac{w_1 / w_0^2}{1 - w / w_0^2} + i \cdot d \cdot \frac{w}{w_0^2} \right), \qquad (14)$$

where $w_1 = 6$Hz, $w_0 = 50$Hz, $d = 1.28$, $A = 10^8$.

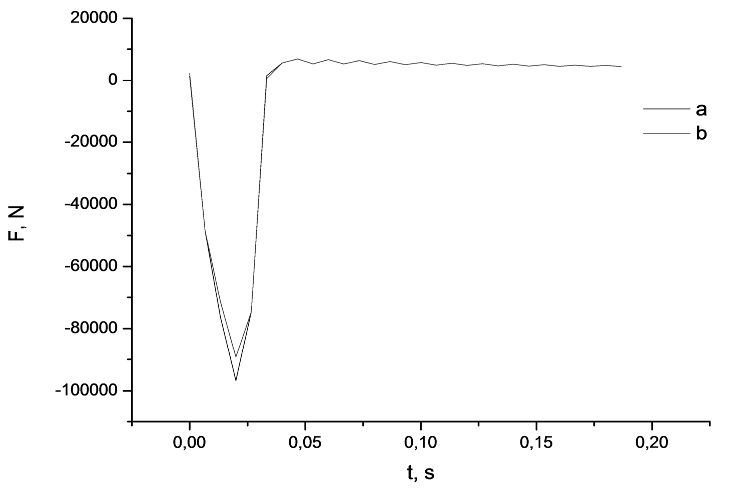

Fig. 4. Comparison of graphs STF (2.5): a – calculated in the inverse problem, b – constructed in the direct problem.

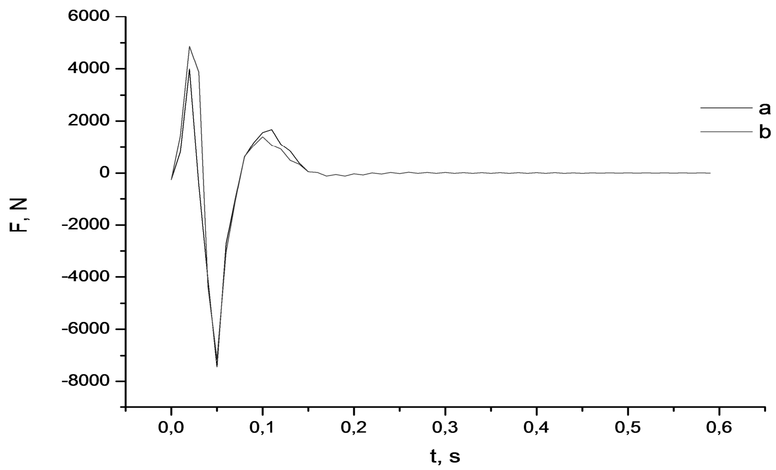

Fig. 5. Comparison of graphs STF preset as a fading sinusoid (14): a – calculated in the inverse problem, b – constructed in the direct problem.

In the third example, the correlation coefficient between a source time function given as a faulting sinusoid and the obtained solution of the inverse problem is equal to 0.9532.

CONCLUSION

In this paper the analytical methods for calculation of the displacement field on the free surface of a layered anisotropic medium (with transversally-isotropic, orthorombic and monoclinic symmetry) are developed, when the source of seismic waves is presented by a randomly oriented force and/or seismic moment tensor. The stress-displacement discontinuity is determined via seismic moment tensor components. For the first time a set of analytical and numerical approaches to determining the earthquake source parameters, based on the direct problem solutions, is proposed. The method of wave fields modelling in layered medium using eigenvectors and eigenvalues is developed.

The method for determining the displacement field on the free surface of an anisotropic inhomogeneous medium from a source presented by a randomly oriented force is tested. Thus, the methods, approaches, algorithms, software for the propagation of seismic waves and results of inverse dynamic problems of seismology proposed and developed by the author and highlighted in the paper, can be successfully used in the study of the seismic regions and effective implementation in the construction of the earthquake source mechanism which is crucial for seismic regions of the country.

Probability and reliability of basic scientific terms and results is provided by well posed problems, rigidity of mathematical methods and transformations in obtaining basic analytical relations for the displacement field and the seismic moment tensor components, by conducting computational experiments with reasonable accuracy, controlled by means of the theoretical relations for variations of physical parameters of studied media and wave forms on the surface of a layered half-space, and is also confirmed by the coincidence with analytical solutions and with results obtained by other methods.

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